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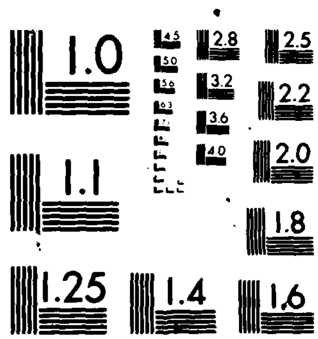

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Let H (t) be the life distribution of a device subject to shocks governed by an integer-valued stochastic process with stationary, independent, nonnegative increments. H (t) is a function of the probabilities P_R of surviving the first k shocks, k = 1,2 .... We show that H (t) inherits various aging properties (IFR, IFRA, NBU) of the discrete survival function P_k. Analogues results hold in the continous case where H(t) is the life distribution of a device subject to wear according to a wear process with stationary, independent nonnegative increments. In the cumulative damage model for the wear process or for the shock

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Shock Models Arising from Processes with Stationary, Independent, Nonnegative Increments

by

Harry Joe and Frank Proschan

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ABSTRACT.

Shock Models Arising from Processes with Stationary, Independent, Nonnegative Increments

by

Harry Joe and Frank Proschan

Let H(t) be the life distribution of a device subject to shocks governed by an integer-valued stochastic process with stationary, independent, nonnegative increments. H(t) is a function of the probabilities $\bar{P}_k$ of surviving the first k shocks, k = 1, 2, ... We show that H(t) inherits various aging properties (IFR, IFRA, NBU) of the discrete survival function $\bar{P}_k$. Analogous results hold in the continuous case where H(t) is the life distribution of a device subject to wear according to a wear process with stationary, independent nonnegative increments. In the cumulative damage model for the wear process or for the shock process, H is IFRA if the process enjoys a TP_2 property.

1. Introduction and Summary.

In Esary, Marshall, and Proschan (1973) [hereafter referred to as EMP (1973)], the life distribution $H(t)$ of a device subject to shocks governed by a Poisson process is considered as a function of the probability $\overline{P}_k$ of surviving the first k shocks, $k = 1, 2, \ldots$. EMP(1973) show that the continuous life distribution $H(t)$ inherits various aging properties of the discrete survival function $\overline{P}_k$ such as IFR, IFRA, NBU, etc.

In Section 2, we extend some of these results to the case in which shocks occur according to an integer-valued stochastic process $\{N(t), t \geq 0\}$ with stationary, independent, nonnegative increments (i.e., a generalized Poisson process). In Section 3, similar results are obtained in the continuous case of a wear process $\{W(t), t \geq 0\}$ with wear occurring according to a stochastic process with stationary, independent, nonnegative increments. In Section 4, we present some examples of well known generalized Poisson processes. Finally, in Section 5, we study cumulative damage models for the shock process $\{N(t), t \geq 0\}$ and the wear process $\{W(t), t \geq 0\}$. In the cumulative damage model for the shock process, the amounts of damage caused by the shocks are independently distributed according to a common distribution, and the device fails when the total accumulated damage exceeds a specified threshold. In this model, the resulting probabilities $\overline{P}_k$ enjoy the discrete IFRA property, and thus the life distribution $H(t)$ of the device is IFRA. In the cumulative damage model for the wear process, the amount of damage caused by a fixed amount of wear always has the same distribution, and, as before, the device fails when the total accumulated damage exceeds a specified threshold. In this model, the resulting probabilities $\bar{F}_u$ of surviving the first u units of wear enjoy the IFRA property (under one assumption) and thus, the life distribution H(t) is again IFRA.

2. Generalized Poisson Process

Let a device be subject to shocks that are governed by a generalized Poisson process. A generalized Poisson process is an integer-valued stochastic process $\{N(t), t \ge 0\}$ with stationary, independent, nonnegative increments (see Parzen, 1962, p. 126). Suppose the probability of the device surviving k shocks is $\bar{P}_k$, where $1 = \bar{P}_0 \ge \bar{P}_1 \ge \bar{P}_2 \ge \cdots$. It follows that the survival function $\bar{H}(t)$ of the device is given by:

$$\bar{H}(t) = \sum_{k=0}^{\infty} \bar{P}_k \, P[N(t) = k]. \tag{2.1}$$

2.1. Definition. A process $\{N(t), t \ge 0\}$ is said to be TP_2 if $P[N(t) = k]$ is TP_2 in $t \in [0, \infty)$ and $k \in \{0, 1, 2, \ldots\}$. (See Karlin, 1968, for the definition of TP_2.)

The following theorem extends a basic result of EMP(1973).

2.2. Theorem. (i) Let $\bar{P}_k$ be discrete NBU ($\bar{P}_{k+m} \le \bar{P}_k \bar{P}_m$ for m, k = 0, 1, 2, ...). Then H given in (2.1) is NBU. (ii) Let $\bar{P}_k$ be discrete IFR ($\bar{P}_k$ is log concave in k = 0, 1, 2, ...) and let $\{N(t), t \ge 0\}$ be TP_2. Then H is IFR. (iii) Let $\bar{P}_k$ be discrete IFRA, i.e., $\bar{P}_k^{1/k}$ is decreasing in k = 0, 1, 2, ... Let $\{N(t), t \ge 0\}$ be TP_2. Then H is IFRA.

Proof. Let $t, s \ge 0$. Then

$$\bar{H}(t+s) = \sum_{k=0}^{\infty} \bar{P}_k \, P[N(t+s) = k] = \sum_{k=0}^{\infty} \bar{P}_k \sum_{m=0}^{k} P[N(t) = m] P[N(s) = k-m]$$

$$= \sum_{m=0}^{\infty} P[N(t) = m] \sum_{k=m}^{\infty} \bar{P}_k \, P[N(s) = k-m]$$

$$= \sum_{m=0}^{\infty} P[N(t) = m] \sum_{k=0}^{\infty} \bar{P}_{k+m} \, P[N(s) = k].$$

The second equality follows from the fact that the increments are stationary and independent.

(i) Since $\overline{P}_k$ is discrete NBU, then

$$\overline{H}(t+s) \le \sum_{m=0}^{\infty} P[N(t) = m) \overline{P}_m \sum_{k=0}^{\infty} \overline{P}_k P[N(s) = k]$$

$$= \overline{H}(t)\, \overline{H}(s),$$

so that H is NBU.

(ii) Since $\overline{P}_k$ is discrete IFR, then $\overline{P}_k$ is log concave in k, or equivalently, $\overline{P}_{k+m}$ is TP_2 in k and -m. Since $P[N(s) = k]$ is TP_2 in k and s, it follows from the Basic Composition Formula (Karlin, 1968, p. 17) that $\sum_{k=0}^{\infty} \overline{P}_{k+m} P[N(s) = k]$ is TP_2 in s and -m. Applying the Basic Composition Formula once more, and the fact that $P[N(t) = m]$ is TP_2 in -m and -t, we conclude that $\overline{H}(t+s)$ is TP_2 in s and -t. Thus $\overline{H}$ is log concave and therefore IFR.

(iii) Let $Q_{N(t)}(\cdot)$ denote the probability generating function of N(t). Then for $0 \le \zeta \le 1$,

$$\sum_{k=0}^{\infty} (\overline{P}_k - \zeta^k) P[N(t) = k] = H(t) - Q_{N(t)}(\zeta)$$

$$= \overline{H}(t) - [Q_{N(1)}(\zeta)]^t = \overline{H}(t) - e^{-t[-\log Q_{N(1)}(\zeta)]}.$$

The second equality follows from the relationship between the probability generating function and the moment generating function. Note also that for $0 \le \zeta \le 1, P[N(1) = 0] \le Q_{N(1)}(\zeta) \le 1$, so that

$$0 \le -\log Q_{N(1)}(\zeta) \le -\log P[N(1) = 0].$$

Since $\overline{P}_k$ is discrete IFRA, then $\overline{P}_k^{1/k}$ is decreasing in k. It follows that $\overline{P}_k - \zeta^k$ changes sign at most once, and if one sign change does occur, it occurs from + to -, for $0 \le \zeta \le 1$. Since $P[(N(t) = k]$ is TP_2, by the Variation Diminishing Property (Karlin, 1968, Chapter 5), $\overline{H}(t) - e^{-\theta t}$ changes sign at most once, and if once, from + to -, for $0 < \theta \le -\log P[N(1)=0]$. Note also that

$$\overline{H}(t) \ge P[N(t) = 0] = Q_{N(t)}(0) = Q^t_{N(1)}(0) = e^{-t\{-\log P[N(1) = 0]\}}$$

It follows that $\overline{H}(t) \ge e^{-\theta t}$ for $\theta \ge -\log P[N(1) = 0]$. Thus $H(t) - e^{-\theta t}$ changes sign at most once, and if once, from + to -, for $\theta > 0$. By a characterization of IFRA distributions (Barlow and Proschan, 1975, p. 89), H must be IFRA. ||

2.3 Remark. In a dual fashion, similar results hold for the dual classes NWU, DFR, DFRA. The NBUE and DMRL classes and their duals have not been successfully treated.

3. Continuous Wear Processes

In this section we obtain analogous results for continuous wear processes. Let $\{W(t),\ t \geq 0\}$ be a stochastic process with stationary, independent nonnegative increments with density function $f_{W(t)}(\cdot)$. Suppose now a device is subject to wear in accordance with this stochastic process. Let $\overline{P}_u$ be the probability of surviving a cumulative wear of u units, with $\overline{P}_u$ satisfying $1 = \overline{P}_0 \geq \overline{P}_u \geq \overline{P}_{u'}$ for $0 < u \leq u'$. We express the survival function $\overline{H}(t)$ of the device as

$$\overline{H}(t) = \int_0^\infty \overline{P}_u\, f_{W(t)}(u)\, du. \tag{3.1}$$

<u>3.1 Definition</u>. The process $\{W(t),\ t \geq 0\}$ is said to be TP_2 if $f_{W(t)}(u)$ is TP_2 in $t,\ u \in [0, \infty)$.

We may now state and prove the continuous analogue of Theorem 2.2.

<u>3.2 Theorem</u>. (i) Let $\overline{P}_u$ be NBU. Then H given in (3.1) is NBU. (ii) Let $\overline{P}_u$ be IFR. Let $\{W(t),\ t \geq 0\}$ be TP_2. Then H is IFR. (iii) Let $\overline{P}_u$ be IFRA and $\{W(t),\ t \geq 0\}$ be TP_2. Then H is IFRA.

<u>Proof</u>. Let $t,\ s \geq 0$. Then $\overline{H}\,(t + s) = \int_0^\infty \overline{P}_u f_{W(t+s)}(u)\, du$

$$= \int_0^\infty \overline{P}_u f_{W(t)} * f_{W(s)}\ (u)\, du$$

$$= \int_0^\infty \overline{P}_u \int_0^u f_{W(s)}(u - x)\ f_{W(t)}(x)\, dx\, du$$

$$= \int_0^\infty f_{W(t)}(x) \int_x^\infty f_{W(s)}(u - x)\ \overline{P}_u\, du\, dx$$

$$= \int_0^\infty f_{W(t)}(x) \int_0^\infty f_{W(s)}(u)\ \overline{P}_{u+x}\, du\, dx.$$

(i) and (ii) now follow as in Theorem 2.1.

(iii) Let $M_{W(t)}(\cdot)$ be the moment generating function of $W(t)$.

Then for $0 < \zeta < \infty$, $\int_0^\infty (\bar{P}_u - e^{-\zeta u}) f_{w(t)}(u)du = \bar{H}(t) - \int_0^\infty e^{-\zeta u} f_{w(t)}(u)du$

$= \bar{H}(t) - M_{w(t)}(-\zeta) = \bar{H}(t) - [M_{w(1)}(-\zeta)]^t = \bar{H}(t) - e^{-t[-\log M_{w(1)}(-\zeta)]}$

For $0 < \zeta < \infty$, we have $0 < M_{w(1)}(-\zeta) \le 1$, so that $0 \le -\log M_{w(1)}(-\zeta) < \infty$. Completing the proof as in Theorem 2.2 (iii), we conclude that H is IFRA. ||

3.3 Remark. As in the discrete case, dual results hold in the continuous case for NWU, DFR, DFRA. Results have not yet been obtained for the NBUE, DMRL classes and their duals.

4. Examples.

A generalized Poisson process (Parzen, pp. 126-127) is an integer-valued stochastic process $\{N(t), t \geq 0\}$ with stationary, independent, nonnegative increments. In a generalized Poisson process, events can occur simultaneously. A generalized Poisson process necessarily has a characteristic function of the form $\phi_{N(t)}(u) = e^{\lambda t[\phi(u)-1]}$ for some $\lambda > 0$ and some characteristic function $\phi(u) = \sum_{k=0}^{\infty} p_k e^{iku}$. This form of characteristic function for $N(t)$ arises when times of possible events follow a Poisson process with intensity λ and the number of events occurring at such times are independently distributed with probability atoms $\{p_k\}$.

If we choose:

(i) $\phi(u) = e^{\nu(e^{iu}-1)}$, the Poisson characteristic function,or

(ii) $\phi(u) = \{p(1-qe^{iu})^{-1}\}^r$, the negative binomial characteristic function, or

(iii) $\phi(u) = (pe^{iu} + q)^n$, the binomial characteristic function,

we obtain a generalized Poisson TP_2 process with corresponding frequency function:

(i) $$P[N(t) = k] = \frac{\nu^k}{k!} e^{-\lambda t} \sum_{j=0}^{\infty} \frac{(\lambda t)^j}{j!} e^{-\nu j} j^k, \text{ or}$$

(ii) $$P[N(t) = k] = q^k e^{-\lambda t} \sum_{j=0}^{\infty} \binom{rj+k-1}{k} \frac{(\lambda t)^j p^{rj}}{j!}, \text{ or}$$

(iii) $$P[N(t) = k] = p^k e^{-\lambda t} \sum_{j=0}^{\infty} \binom{nj}{k} \frac{(\lambda t)^j}{j!} q^{nj-k}.$$

An example of a stochastic process $\{W(t), t \geq 0\}$ with stationary, independent nonnegative increments such that the density $f_{W(t)}(u)$ is TP_2 in t, $u \in [0, \infty)$ is the gamma process:

$$f_{W(t)}(u) = \lambda^{\alpha t} u^{\alpha t-1} e^{-\lambda u}/\Gamma(\alpha t), \quad \alpha > 0.$$

5. Cumulative Damage

We next consider a model in which wear causes damage, damages cumulate, and the device fails when a specified cumulative damage threshold is exceeded. We assume that the amount of damage caused by a fixed amount of wear always has the same distribution. Let the amount of damage caused by one unit of wear have distribution F, infinitely divisible and satisfying $F(0^-) = 0$. Let the damage threshold be denoted by x, and the probability of surviving u units of wear be denoted by $\overline{P}_u$. Then $\overline{P}_u = F^{(u)}(x)$, defined more precisely below. We shall show that under an appropriate assumption, $\overline{P}_u^{\,1/u} = [F^{(u)}(x)]^{1/u}$ is decreasing in $u \geq 0$ for fixed x, so that $\overline{P}_u$ has the continuous IFRA property.

Let $\lambda(\cdot)$ be the unique complex-valued function of t with $\lambda(0) = 0$ that is continuous at 0 satisfying $\phi(t) = e^{\lambda(t)}$ (see Chung, p. 241), where ϕ is the characteristic function corresponding to F. Since ϕ is an infinitely divisible characteristic function, $\phi^r(\cdot) = e^{r\lambda(\cdot)}$ is a characteristic function for all $r \geq 0$ (see Parzen, p. 124). Let $F^{(r)}$ be the distribution function with characteristic function ϕ^r. Then

$$F^{(r)} * F^{(s)} = F^{(r+s)}, \quad r,\ s \geq 0.$$

5.1 Theorem. Let x be a continuity point of $F^{(r)}$ for all $r \geq 0$. Then $[F^{(r)}(x)]^{1/r}$ is decreasing in $r \geq 0$.

Proof. For an arbitrary distribution function G with $G(0^-) = 0$, $[G^{(n)}(x)]^{1/n}$ is decreasing in positive integer n for each fixed $x \geq 0$ (see Barlow and Proschan, 1975, p. 94), where $G^{(n)}$ is the n-fold convolution of G. Let $r_1 < r_2$ be positive rationals, $r_1 = \frac{m_1}{n}$, $r_2 = \frac{m_2}{n}$, $m_1 < m_2$, m_1, m_2, n are positive integers. Then applying the referenced theorem with $G = F^{(1/n)}$, we have:

$$\{(F^{(1/n)})^{(m_2)}(x)\}^{1/m_2} \le \{(F^{(1/n)})^{(m_1)}(x)\}^{1/m_1} \text{ or}$$

$$\{F^{(m_2/n)}(x)\}^{n/m_2} \le \{F^{(m_1/n)}(x)\}^{n/m_1} \text{ or}$$

$$\{F^{(r_2)}(x)\}^{1/r_2} \le \{F^{(r_1)}(x)\}^{1/r_1}$$

Therefore, $\{F^{(r)}(x)\}^{1/r}$ is decreasing in r over the positive rationals for each fixed $x \ge 0$.

Now suppose x is a continuity point of $F^{(r)}$ for each $r \ge 0$. Let r and s be positive reals, $r < s$. Let $\{r_n\}$ be a sequence of rationals such that $r_m \downarrow r$ and $r_n < s$ for all n; let $\{s_n\}$ be a sequence of rationals such that $s_n \downarrow s$. Then $r_n \le s_n$ for all n implies that $[F^{(r_n)}(x)]^{1/r_n} \ge [F^{(s_n)}(x)]^{1/s_n}$ for all n. By the Continuity Theorem,

$F^{(r_n)}(x) \to F^{(r)}(x)$ and $F^{(s_n)}(x) \to F^{(s)}(x)$. Thus $[F^{(r_n)}(x)]^{1/r_n}$ $\to [F^{(r)}(x)]^{1/r}$ and $[F^{(s_n)}(x)]^{1/s_n} \to [F^{(s)}(x)]^{1/s}$, with $[F^{(r)}(x)]^{1/r} \ge$ $[F^{(s)}(x)]^{1/s}$. Thus we conclude $\{F^{(r)}(x)\}^{1/r}$ is decreasing in $r \ge 0$. ||

Using the IFRA property of $\overline{P}_u$ and of $\overline{P}_k$, we obtain the following theorems.

<u>5.2 Theorem</u>: Let $\overline{H}(t) = \int_0^\infty F^{(u)}(x) f_{w(t)}(u)\,du$ represent the survival probability in the cumulative damage model for the wear process $\{w(t), t \ge 0\}$. Suppose x is a continuity point of $F^{(u)}$ for all $u \ge 0$ and suppose $\{w(t), t \ge 0\}$ is TP_2. Then H is IFRA. ||

<u>Proof</u>: This follows from theorem 5.1 and Theorem 3.2 iii.

<u>5.3 Theorem</u>. Let $\overline{H}(t) = \sum_{k=0}^{\infty} G^{(k)}(x) P(N(t) = k)$ represent the survival probability in the cumulative damage model for the shock process $\{N(t), t \ge 0\}$, where G is the distribution function for damage due to a single shock $(G(0^-) = 0)$ and $G^{(k)}$ is the k-fold convolution of G. Suppose $\{N(t), t \ge 0\}$ is TP_2. Then H is IFRA.

<u>Proof</u>. This follows from Theorem 2.2(iii) and the fact that $[G^{(k)}(x))^{1/k}$ is decreasing in k for each $x \ge 0$. ||

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